Determination of critical current density in bulk melt-processed high temperature superconductors from levitation force measurements

A. A. Kordyuk,[a] V. V. Nemoshkalenko, and R. V. Viznichenko
Institute of Metal Physics, Kyiv, Ukraine

W. Gawalek and T. Habisreuther
Institut für Physikalische Hochtechnologie, Jena, Germany

(Received 26 April 1999; accepted for publication 19 July 1999)

A simple approach to describe levitation force measurements on bulk melt-processed high temperature superconductors was developed. A couple of methods to determine the critical current density J_c were introduced. The averaged a-b-plane J_c values for the field parallel to this plane were determined. The first and second levitation force hysteresis loops calculated with these J_c values coincide remarkably well with the experimental data. © *1999 American Institute of Physics.* [S0003-6951(99)05037-8]

Superconducting systems with magnetic levitation have long been known and the discovery of high temperature superconductors (HTS) highly stimulated their investigation, but a real interest in them for large scale applications appears only with the successful development of the melt-processed (MP) technology.[1] The use of MP HTS in large scale systems such as flywheels for energy storage, electric motors and generators, permanent magnets, etc. is the most promising HTS application now.[2] In this applied region the levitation force measurements can be considered in two roles: as an information source to know more about levitation systems and as a quick technique to test HTS samples.[3] In many earlier works[4,5] it has been shown that the forces between a PM and HTS sample are closely related with HTS magnetization curves. Vertical levitation force versus vertical distance $F_z(z)$ is the nearest analog to $M(H)$ dependencies with their major and minor hysteresis loops, but the complexity of a field configuration in such large scale PM-HTS systems makes it very difficult to directly correlate them in general. The problem can be solved by numerical approaches,[6] but this usually needs too much computer resources to be applicable to direct HTS sample investigation. To perform such an investigation an analytical evaluation is more wished for.

Two limiting cases of HTS structure have been considered as analytical to calculate the dynamic parameters of an idealized system of a point magnetic dipole over an infinite flat superconductor. The first one is the ''granular superconductor'' which can be modeled by a set of small isolated superconducting grains.[7] The second one is an ''ideally hard superconductor.''[8–11] It was shown recently[8–10] that dynamics in a wide variety of levitation systems can be described in terms of surface screening currents which screen alternating magnetic field component due to PM displacements, as it would be for an ideally hard superconductor, i.e., a superconductor with infinite pinning forces. For an infinite flat superconductor the frozen-image method was introduced[8] as an illustration of simple analytical calculation of forces acting in the system. Good agreements with experiments were found for PM resonance oscillations frequencies[9] and, recently, by Hull and Cansiz,[12] for both vertical and lateral force components.

The feasibility of the ideally hard superconductor approach is that the penetration depth δ of alternating magnetic field is much less than system dimensions.[8,9] To calculate the stiffness or resonance frequencies the limit $\delta \to 0$ can be used, but it was shown that taking into account the finite values of δ it is possible to calculate the ac loss[10] and even recover critical current density profiles within δ depth from ac loss measurements.[11] In this letter we present such an approach of levitation force calculation (including its hysteresis behavior) for a superconductor with finite values of critical current density J_c and simple methods to obtain J_c values from levitation force experimental data.

A PM placed over ideally hard superconductor induces at its surface the screening currents $\mathbf{j} = (c/4\pi)\mathbf{n} \times \mathbf{b}_r$, where $\mathbf{n}$ is the surface normal and $\mathbf{b}_r$ is the tangential magnetic field component at the surface (the normal component b_n at the surface is zero). From the symmetry, for an infinite flat surface[9]

$$\mathbf{b}_r = 2\mathbf{b}_{ar}, \tag{1}$$

where $\mathbf{b}_a$ is the variation of the PM magnetic field $\mathbf{B}_a$ due to its displacements in respect to initial field cooled (FC) position: $\mathbf{b}_a = \mathbf{B}_a - \mathbf{B}_{a\mathrm{FC}}$.[9] For the z-axial symmetric configuration $\mathbf{r} = (r \sin\theta, r\cos\theta, z)$ where only j_θ component is induced, for the vertical force acting on PM from the screening currents one can write

$$F_{id} = \int_0^\infty r b_{ar}^2(r)\, dr. \tag{2}$$

This is an ideal force, which can be readily calculated just from the known tangential component of PM field. Equation (2) is obtained from zero-depth screening currents approach that we will call a zero approximation of real PM-HTS systems. Within this approximation any configuration of such systems can be calculated numerically[13] but to describe hysteresis phenomena next-order approximations have to be considered.

[a]Electronic mail: kord@imp.kiev.ua

In the second stage (a first approximation) we will examine a model where: (i) δ is finite but still much less than system dimensions L, (ii) the critical state model is applicable to these samples, and (iii) critical current density is constant. The applicability of the critical state model to melt-processed HTS has been proven in many experiments[3,10,11] and is quite acceptable here. The first condition on δ can be written as

$$\delta(r) \ll B_{ar}(r)\left(\frac{dB_{ar}(r)}{dr}\right)^{-1} \sim L, \tag{3}$$

and, because $\delta \propto b_r$, Eq. (3) can always be satisfied by limiting the minimum distance between PM and HTS surface. One can estimate $L \approx z + d/2$, where z is, here and below, the distance between PM and HTS surface and d is the PM thickness.

J_c usually depends on both magnetic field and space coordinate, but, as will be shown below, we can accept the constant J_c condition for levitation force measurements. This just means that in the next relation for j, the surface density of screening currents,

$$j(r,z) = \delta(r,z) J_c = \frac{c}{2\pi} b_{ar}(r,z), \tag{4}$$

J_c can be treated as a coefficient between j and δ, a coarse-grained flux penetration depth averaged over L scale. The $j(r)$ function does not depend on field history but only on PM position z in the same way as $b_{ar}(r)$, the distribution at the HTS surface of the PM field variation, that after cooling is a function of r and z. Thus, in this approximation, the function $\delta(r,z)$ formally does not depend on field history but means the flux penetration depth at the first PM descent only.

Next, if we use a protocol of PM motion according to which it moves between two points: the initial or FC point $z_{\max}$ that is included in $b_{ar}(r,z)$ function as a condition $b_{ar}(r,z_{\max})=0$ ($z_{\max}=\infty$ for ZFC case), and the lowest point $z_{\min}$, the current distributions in the depth z of superconductor are the following. After the PM stops the first descent and begins to go up (the first ascent), the depth of the layer where currents flow remains constant and is equal to its maximum value $\delta_{\max} \equiv \delta(r,z_{\min})$, but there are two regions with opposite currents. The opposite flowing current penetrates from the top at the depth $\delta_\uparrow$ that can be obtained from Eq. (4)

$$\delta_\uparrow(r,z,z_{\min}) = \frac{1}{2}[\delta(r,z_{\min}) - \delta(r,z)]. \tag{5}$$

Its maximum value is $\delta_{\uparrow\max} \equiv \delta_\uparrow(r,z_{\max},z_{\min}) = \delta(r,z_{\min})/2$, so during the second descent there are three regions with $+J_c$ for $0<\zeta<\delta_\downarrow$, $-J_c$ for $\delta_\downarrow<\zeta<\delta_{\uparrow\max}$, and $+J_c$ for $\delta_{\uparrow\max}<\zeta<\delta_{\max}$, where $\delta_\downarrow(r,z)=\delta(r,z)/2$ also does not depend on $z_{\min}$. If one can neglect flux creep for times greater than the descent-ascent time, any other ascents are equal to the first one and any other descents are equal to the second one. Any other current distributions for other protocols, for example to describe minor hysteresis loops, can also readily be obtained within the scheme above.

Applying this scheme to calculate the vertical forces during the first descent $F(z)$, the first and the next ascents $F_\uparrow(z,z_{\min})$ and the second and the next descents $F_\downarrow(z,z_{\min})$ one can write

$$F(z) = \frac{2\pi}{c} J_c \int_0^\infty r\,dr \int_0^{\delta(r,z)} d\zeta\, b_{ar}(r,z+\zeta), \tag{6}$$

$$F_\uparrow(z,z_{\min}) = \frac{2\pi}{c} J_c \int_0^\infty r\,dr \left[\int_{\delta_1}^{\delta_{\max}} d\zeta - \int_0^{\delta_\uparrow} d\zeta\right] \times b_{ar}(r,z+\zeta), \tag{7}$$

$$F_\downarrow(z,z_{\min}) = \frac{2\pi}{c} J_c \int_0^\infty r\,dr \left[\int_{\delta_{\max}/2}^{\delta_{\max}} d\zeta - \int_{\delta/2}^{\delta_{\max}/2} d\zeta + \int_0^{\delta/2} d\zeta\right] b_{ar}(r,z+\zeta). \tag{8}$$

The functions $\delta(r,z)$ depend on J_c according to the above equations [Eqs. (4), (5) and below] and for $J_c \to \infty$ all these forces become equal to $F_{id}(z)$.

Remaining within the condition (3) we can approximate the integrals over z from the formulas (6)–(8) by multiplying the depth of the layer where current flows by the field bar in its center. It is easy to show that within the above approximation the formula (6), for example, can be rewritten as

$$F(z) = \int_0^\infty r\, b_{ar}(r,z)\, b_{ar}\left(r, z+\frac{\delta}{2}\right) dr, \tag{9}$$

which highly increases the calculation speed.

To check the applicability of the above consideration to real MP HTS we used a standard experimental setup on levitation force measurements.[3] The $SmCo_5$ disk shape PM was 15 mm in diameter and 8 mm in thickness (the effective thickness with ferromagnetic holder that was evaluated from real PM field configuration was 12.7 mm) with averaged axial magnetization of $4\pi M = 9236$ G (the field measured by Hall probe in its center at the distance of 0.8 mm from its bottom surface was 3350 G). The magnetic field of the PM was calculated as field of a coil with the same dimensions and with lateral surface current density $J = cM$. All measured samples were melt-processed HTS of 30 ± 0.5 mm in diameter and 17.5 ± 0.5 mm in thickness. The distance z between PM bottom surface and HTS top surface varied from $z_{\max} = 400$ mm (that can be considered as ZFC case) to its minimum value $z_{\min} = 0.5$ mm. The minimum step of PM motion was 75 μm. The accuracy of force detecting was 15 mN. Within this accuracy the experimental data were reproducible for every sample. Figure 1 represents the first and second hysteresis loops (the first and second descent and ascent) for two samples.

Within the above approximation we have only one parameter, J_c, for forces of Eqs. (6)–(8) [or Eq. (9) and analogous ones] to be fitted to the experimental ones $F_{\exp}(z)$. To do this, we have to choose one of these functions and one point z_i, and solve the equation

$$F(J_c, z_i) = F_{\exp}(z_i). \tag{10}$$

The forces calculated from formulas (6)–(8) with the J_c values obtained from Eq. (10) in $z_{\min}$ point are also repre-

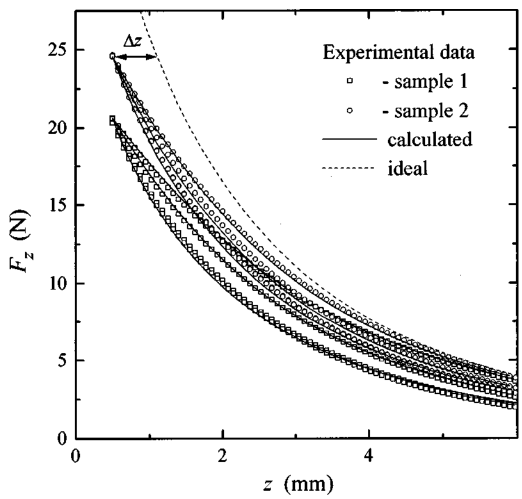

FIG. 1. Experimental (symbols) and calculated (solid lines) data on the first and second hysteresis loops of the vertical levitation force vs distance z between PM and HTS surface. Dashed line represents the force for an ideal superconductor.

sented in Fig. 1 by solid lines. The forces calculated from Eq. (9) and from analogous ones practically coincide with the above in the $F(z)$ plot scale. A good agreement between the experimental and calculated $F(z)$ dependencies demonstrates the above approximation is correct.

Nevertheless, discrepancy between the experimental and calculated forces still exists and is larger than the experimental accuracy. One of the most likely reasons is a variation of J_c with depth and field. Figure 2 shows the values of J_c versus maximum value of $B_r(z)$ at the HTS surface for two HTS samples. The data were obtained by solving Eq. (10). Open symbols represent the solution for the function of Eq. (6), and solid symbols represent the solution for the function of Eq. (9). The solid line in Fig. 2 with respect to the right axis represents the dependence of $B_{r\,\mathrm{max}}(z)$. For a perfectly uniform sample with c axis exactly perpendicular to the surface such a dependence of $J_c(B_{r\,\mathrm{max}})$ would be uniquely determined by the dependence of $J_c^{ab}(B_{ab})$, the critical current density flowing in a-b-plane versus the magnetic field parallel to this plane. But for real melt processed samples, it is more reasonable to assume that the dependencies of $J_c(B_{r\,\mathrm{max}})$ in Fig. 2 are mostly caused by space variations of critical current density. The steep slope of the curves in Fig. 2 at low field, which caused the maximum in the upper curve, is related with the finite diameter of the HTS samples and shows the lower field limit for the given configuration.

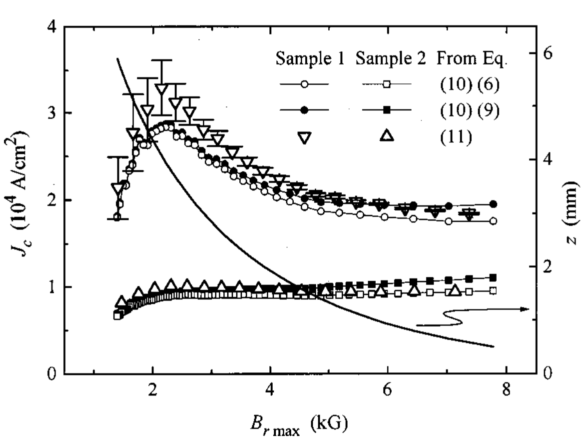

FIG. 2. The values of averaged critical current density vs maximum value of B_r, the magnetic field tangential component at the HTS surface, obtained by different methods. The solid line with respect to right axis represents the dependence of $B_{r\,\mathrm{max}}(z)$.

There is a possibility here to introduce a visually simple method to evaluate J_c. It is understandable that in the spirit of the above consideration a shift Δz of the first descent experimental curve with respect to the calculated ideal one (Fig. 1) has to be proportional to an average penetration depth. From the condition $F_{id}(z+\Delta z)=F(J_c,z)$ and Eq. (9) one can readily obtain

$$\delta \approx 4\Delta z, \quad \text{or} \quad J_c \approx \frac{c}{8\pi}\frac{b_{ar}}{\Delta z}. \tag{11}$$

The values of J_c evaluated in such a way are also represented in Fig. 2 and show a good agreement with ones determined before. The experimental error σ_{J_c} that is shown here was estimated from the formula $\sigma_{J_c}/J_c = \sigma_F(dF/dz)^{-1}/\Delta z$ which assumes the maximum error is caused by the force measurement: $\sigma_F \approx 30$ mN.

In summary, we have considered the approach, which we call the ''first approximation,'' to describe levitation force data. The term ''first'' implies that we consider a case in which such parameters as flux penetration depth δ or normal component of magnetic field at HTS surface b_n are not zero, as it is for an ideally hard superconductor,[8] but small enough: $\delta \ll L$, $b_n \ll b_r$. Within this condition the methods to calculate J_c, the critical current density, which we have introduced in the letter are exact. Remarkably, the approach works well even beyond this condition, when $\delta \sim L$, $b_n \sim b_r$. In this region the methods become empirical. The J_c value that can be obtained by the methods is averaged over L scale critical current density in a-b-plane for field parallel to this plane: $J_c = \langle J_c^{ab}(\mathbf{B} \| ab)\rangle_L$. L scale depends on the size of a magnet we use.

The authors would like to thank J. R. Hull for helpful discussions and T. Strasser for assistance with the experimental setup.

[1] D. A. Cardwell, Mater. Sci. Eng., B **53**, 1 (1998).
[2] J. R. Hull, IEEE Spectr. **34**, 20 (1997).
[3] G. Fuchs, P. Stoye, T. Staiger, G. Krabbes, P. Schatzle, W. Gawalek, P. Gornert, and A. Gladun, IEEE Trans. Appl. Supercond. **7**, 1949 (1997).
[4] E. H. Brandt, Appl. Phys. Lett. **53**, 1554 (1988).
[5] F. C. Moon, K.-C. Weng, and P.-Z. Chang, J. Appl. Phys. **66**, 5643 (1989).
[6] T. Sugiura, H. Hashizume, and K. Miya, Int. J. Appl. Electromagn. Mater. **2**, 183 (1991).
[7] A. A. Kordyuk and V. V. Nemoshkalenko, Appl. Phys. Lett. **68**, 126 (1996).
[8] A. A. Kordyuk, J. Appl. Phys. **83**, 610 (1998).
[9] A. A. Kordyuk, Metal. Phys. Adv. Tech. **18**, 249 (1999).
[10] A. A. Kordyuk, V. V. Nemoshkalenko, R. V. Viznichenko, and W. Gawalek, Mater. Sci. Eng., B **53**, 174 (1998).
[11] A. A. Kordyuk, V. V. Nemoshkalenko, R. V. Viznichenko, and W. Gawalek, Physica C **310**, 173 (1998).
[12] J. R. Hull and A. Cansiz (unpublished).
[13] V. V. Vysotskii and V. M. Pan, Inst. Phys. Conf. Ser. **158**, 1667 (1997).